Guaranteed vs. Direct Lending: The Case of Student Loans

Deborah Lucas, Northwestern University and NBER
Damien Moore, Congressional Budget Office

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1. Introduction

The federal government makes low-cost financing for higher education widely available through its fast-growing student loan programs. The existence of two competing government programs provides a unique opportunity to compare the cost to the government of direct federal lending versus loan guarantees. It also offers insights into the political economy of the federal budget process.

Both the direct and guaranteed student loan program offer borrowers very similar loan products and terms. They differ significantly, however, from the perspective of other key stakeholders, including educational institutions, commercial lenders, and state guarantee agencies. The programs also have widely divergent budget costs: The FY2007 budget records a 2 percent subsidy rate on direct loans, versus a subsidy rate of 10 percent for the guaranteed program.

In this study, we propose and implement a methodology to estimate the cost of the two programs in market value terms. In doing so, we address the question of how much of the difference in reported subsidy rates can be attributed to real cost differences, and how much is due to idiosyncrasies in the rules for budgeting for federal credit. A potential source of a real cost differential is in the cost of capital. Data from the secondary market for guaranteed student loans provides some interesting evidence on the size of this effect.

To preview the main results, we find that budget costs for both programs are well below their market value. This is mostly attributable to budget rules that require discounting expected net cash flows at Treasury rates. Understatement of the market cost of capital also accounts for why some direct loans appears to make money for the government, despite the favorable terms offered to borrowers. Administrative costs are accounted for inconsistently across programs, complicating cost comparisons. Nevertheless, it appears that the guaranteed program is fundamentally more expensive than the direct program. Guaranteed lenders are paid more than is required to induce them to lend at statutory terms. The excess funds are largely absorbed in competition for borrowers, which occurs through various discounts, marketing activities, and higher service levels and subsidies to educational institutions. To the extent that the market is not perfectly competitive, guaranteed lenders presumably retain some of the surplus as profit.

The rest of the paper is organized as follows. Section 2 provides an overview of federal student loan programs: their size, product offerings, the roles of various stakeholders, and their market structure. Section 3 describes how student loans and loan guarantees are budgeted for, how these rules have influenced structural changes in the program over time, and the decomposition of costs in the budget. In Section 4 we discuss the private student market, and the information it provides on the market cost of capital and administrative costs.

In Section 5 we turn to the central problem of estimating the market value of federal student loans and loan guarantees. The exercise requires modeling loan cash flows, which under the direct loan program are affected by defaults, prepayments, forbearance,
deferrals and other embedded options. For the guaranteed program, government cash flows are also affected by payments to and fees from private lenders. Identifying financing versus administrative costs is important both for assessing total cost, and for understanding the cost differential between programs. Rates on private student loans are used as a starting point for risk adjustment. The resulting market value estimates are presented and subjected to sensitivity analysis. Section 6 concludes with a discussion of some of the broader policy questions raised by the analysis.

2. Overview

The Department of Education (ED) oversees two competing student loan programs: the Federal Family Education Loan (FFEL, or guaranteed) program, and the William D. Ford Federal Direct Loan (Direct Loan) program. In the guaranteed program, which dates back to the mid-1960s, the government guarantees loans originated by private lenders against losses from default and makes supplemental payments to lenders. In the direct program, which began operation much more recently in 1994, the government directly lends to qualifying students.

2.1 Program Size

The federal student loan program is one of the largest credit programs operated by the U.S. government. Table 1 shows the rapid growth in total federally-backed student loans outstanding, which in 2005 totaled over $380 billion. Statistics compiled by the Department of Education indicate that in the same year, about 6.8 million students, and 750,000 parents of students, borrowed $56 billion in new federally backed loans (and an additional $69.6 billion of old loans were consolidated). The guaranteed program was responsible for 77 percent of this new loan volume. Another 2.5 million borrowers took advantage of the option to convert their outstanding Stafford loans into more favorable consolidation loans.

2.2 Product Offerings

Both programs offer three basic types of loans, with loan terms set by statute under the Higher Education Act:

**Stafford.** These 10- to 30-year loans are available to students enrolled in eligible educational institutions, which includes most U.S. colleges and universities but not trade or for-profit schools. Between 1998 and July 2006, these loans carried a floating rate that reset annually, based on the 3-month Treasury rate plus a fixed spread. Starting in July 2006, Stafford loans carry a fixed 6.8 percent per annum interest rate, with flexible repayment plans that begin upon completion or dropping out of a course of study. Stafford loans may be “subsidized” or “unsubsidized,” the difference being that the federal government pays all of the accrued interest on subsidized Stafford loans while a borrower is in school, grace or deferment, whereas the interest accrues on unsubsidized loans. Both types carry a below-market interest rate. Eligibility for subsidized loans is based on income. Borrowers are assessed a one time 3 percent origination fee, although
this may be paid by the lender in the guaranteed program or reduced in the direct program. Recent legislation gradually phases out the origination fee.

**Parent Loan for Undergraduate Students (PLUS).** Traditionally, these were loans made to parents of students attending eligible institutions. Starting in July 2006, PLUS borrowers will pay a fixed 8.5 percent per annum rate, and students attending graduate schools became eligible to take out PLUS loans. For parent borrowers, loan repayment begins immediately, whereas student borrowers begin repayment if their course load drops below 50 percent of full time status. All PLUS loans are unsubsidized, but still carry interest rates generally below those in private credit markets.

**Consolidation.** Borrowers with one or more Stafford loans may replace them with a single consolidation loan any time after completing their course of study. Consolidation loans offer a new repayment plan and an interest rate equal to the weighted average of interest rates on the underlying Stafford loans rounded up to the nearest eighth of a percentage point. Thus, the portion of post July 2006 Stafford loans that are consolidated will carry a rate slightly above 6.8 percent. Consolidation loans now offer a similar set of flexible repayment terms as Stafford loans, but in the past also provided additional repayment extension options.

Although guaranteed lenders receive lower compensation from the government for Consolidation than for Stafford loans the returns are still positive, and competition to offer these loans is brisk. An exception is that guaranteed lenders often avoid consolidating distressed loans, which are more expensive to administer and have a lower expected income stream. As a result, a disproportionate share of distressed loans is consolidated into the direct loan program.

### 2.3 Stakeholders

Students and parents of students pursuing post-secondary degrees clearly benefit from these programs, which lower the cost and increase the availability of funding for higher education. From an economic perspective, such assistance can be welfare improving when imperfections in private credit markets limit access to education, or when education has significant positive externalities. A caveat is that some students appear to borrow excessively to pay for degrees that add little to their earnings potential. Unsubsidized Stafford loans are not means-tested, and borrowing limits are tied to educational expenses, which are higher at more expensive schools. Hence, middle and upper-middle class students receive a large share of program benefits.

Educational institutions also depend on federal student loan programs for financial support. Without assistance, many students would be unable or unwilling to pay the high

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1 Several studies question the effectiveness of such policies, for example, De Fraja (2002), Dynarski (2002), Edlin (1993), Hanushek (1998), and Keane (2002). Gale (1991) points out that many federal credit programs probably have a small real effect on the allocation of credit, in many cases simply crowding out private borrowing and lending.
tuition charges at many schools. To a lesser extent, schools benefit directly if they elect
to participate in the guaranteed loan program. Guaranteed lenders offer schools various
types of support in exchange for featuring their loans, including educational grants and
administrative, educational, and systems support to financial aid offices. In “school as
lender” programs, where the educational institution itself takes on the origination role, the
school retains the excess of government payments over its cost of extending credit.

Providing financing for guaranteed student loans have been a profitable line of business
for private lenders, although competition in the industry has intensified over time and
with the liberalization of interstate banking. Over 3,500 lenders originate, service and
finance federally guaranteed loans. The market is dominated by a few large lenders,
including the leading commercial banks and Sallie Mae. Sallie Mae, by far the largest
guaranteed lender, began as a government sponsored enterprise but now is fully
privatized.

State and private nonprofit guaranty agencies are another constituency that benefits
financially from the guaranteed loan program. These entities administer the federal
guarantee and provide services to schools and lenders. As of 2006, there were 35 active
guaranty agencies, some operating in multiple states. Each guarantee agency maintains
an account in federal trust, which is used to pay out claims from lenders. Those funds are
replenished by the federal government. Guaranty agencies also receive federal funds for
performing collection activities, historically as high as 25 percent of the recovered
amounts (even if the amount was recovered through federal loan consolidation). The
agencies may use their share of collections to fund scholarships, education outreach
programs and for default aversion activities.

2.4 Market Structure

Schools have a choice of whether to participate in the direct or guaranteed loan program.
Simultaneous participation in both programs is not permitted, but a school can elect to
switch programs and some choose to do so. Competition for volume between the two
programs therefore centers on school administrators, particularly financial aid officers.
Recall that both programs offer students nearly identical loan terms, so differentiation
occurs along other dimensions. The direct program offers greater administrative
simplicity, which initially attracted many schools to the program. Guaranteed lenders
responded by offering schools and borrowers improved service and other inducements,
and since the late 1990s the guaranteed program has slowly regained market share (see
Figure 1).

Competition also takes place at each FFEL school between guaranteed lenders. Although
there are thousands of lenders potentially competing for borrowers’ business, at the
school level competition is much more limited. The financial aid office serves as a
gatekeeper, counseling students who seek advice, and only including a limited number of

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2 Some have argued that the generous borrowing limits in the federal student loan program have
accommodated the growth in college tuition, which has exceeded the growth of the overall economy. (Add
citation)
3 A single university may have some schools participating in the direct program and others using
the guaranteed program.
lenders on its “preferred lender list.” Most students have little financial experience and rely on the advice of the school, although direct-to-student marketing of loan products is becoming more common and some students venture beyond the preferred lender list. Northwestern University provides a fairly typical example close to home. It includes five major lenders on its preferred list for undergraduate students. It does not officially rank them, but Citibank holds the coveted first position on the (non-alphabetical) list. The preferred lender lists for Northwestern’s various graduate and professional schools are shorter. The business, law and medical schools offer only three options, with the first one being Northwestern University itself. Only two lenders are recommended to students pursuing part-time MBAs.

Understanding the competitive structure of guaranteed lending is important for identifying the likely disposition of rents arising from federal payments to lenders in excess of production costs. Although we do not model these interactions formally, it is reasonable to believe that competition for access to borrowers leaves guaranteed lenders with little in the way of abnormal profits. The gatekeeper role of schools suggests that they capture a large portion of rents, but those may be passed through to students through scholarships, expanded program offerings, or other means. The common practice of lenders paying the origination fee for students is evidence that some of the rent goes directly to students. Some rents are also absorbed by marketing costs and inducements to schools and financial aid officers that are unlikely to provide much benefit to students.

3. Budget Estimates

Most analyses of the cost difference between the direct and guaranteed loan programs rely on budget estimates prepared by the Congressional Budget Office (CBO) and the Office of Management and Budget (OMB). Budget estimates are systematically lower than the market value costs described in Section 4. A premise underlying this study is that market values offer a more accurate representation of the cost of credit to the government than do current budget estimates. Perhaps the strongest argument for adopting this view that almost all non-credit transactions -- including grants, purchases of goods, and the direct provision of services -- appear in the budget at market prices (see Lucas and Phaup (2007) for a more complete discussion). In the context of improving access to higher education, outright grants are an alternative to student loans. The omission of the cost of risk on the loan or loan guarantee artificially favors expanding the student loan programs for education over funding for grants. Nevertheless, the budget provides a useful starting point for evaluating program cost. In this section we describe the rules governing budgeting for credit, how changes in budget rules over time have affected the federal student loan program, and what budget estimates reveal about the breakdown of cost for the two programs.

3.1 Budgeting for Federal Credit Programs

Before 1990, credit, like most other government programs, was accounted for on a cash basis. For a new direct loan program this implied a large up-front cost equal to the principal borrowed, with no offset for expected future repayments. An economically
equivalent credit guarantee program had a much lower, or even negative up-front budget cost. For credit guarantees, few defaults occur in the first year and guarantee fees are collected up front. This accounting favored new guarantee programs over almost all alternative policies, including new direct loan programs.

The Federal Credit Reform Act of 1990 (FCRA) effectively put credit on an accrual basis, with cost measured as the net present value of current and future cash flows associated with the current year commitment. Although it went a long way toward making credit and other forms of assistance comparable, it fell short of measuring cost in terms completely equivalent to cash spending. The biggest discrepancy arises from the mandated use of interest rates on maturity-matched U.S. Treasury securities for discounting, rather than a market-based cost of capital that includes the cost of market risk. It also treats administrative costs inconsistently across programs, with some costs included in subsidy rates and others recorded elsewhere on a cash basis.

3.2 Effects of Budget Rules on Student Loans

Accounting conventions have had a significant effect on the structure and evolution of the federal student loan programs. Most notably, the direct student loan program appears to have been made feasible from a budgetary perspective by FCRA. Proposed on several occasions in the late 1980’s, its high initial cash cost was a decisive obstacle. The direct loan program was enacted in 1993 shortly after the FCRA went into effect.4

As for other credit programs, the mandated use of maturity-matched Treasury rates without risk adjustment, and the inconsistent treatment of administrative costs, drive a wedge between budget estimates and the market value estimates of program cost. The inconsistent treatment of costs across programs is particularly pronounced for student loans. Unlike for many credit programs, administrative costs are included in the subsidy rate reported for guaranteed loans. This occurs because the federal government makes supplementary allowance payments to private lenders to cover the lender’s administrative and other expenses in excess of the amounts collected from borrowers. Inasmuch as those payments continue for the life of the guaranteed student loan, they are capitalized and included in subsidy costs. By contrast, administrative costs in the direct program are accounted for separately on a cash basis, and not included in subsidy estimates. The extent to which this inconsistency affects the difference in estimated subsidy rates has been estimated to be no more than 1.5 percentage points (CBO, 2005).

A discrepancy between budget and market value cost also arises from the budgetary treatment of floating rate loans. The only large federal direct lending program with floating rates was the direct student loan program through July 2006. The FCRA is interpreted as requiring the use of long-term Treasury rates for discounting, whereas market values reflect the shorter effective maturity of floating rate loans. Because of the term premium in long-term rates, this tends to bias down federal estimates of direct loan value. This undervaluation has potential real effects. For instance, it prompted the

4 It is often said that the FCRA was viewed as the direct student loan enabling act.
Department of Education to propose a sale of direct loans in 2003. The plan was to sell the loans, apply some of the proceeds to paying off Treasury debt, and to use the net gain to provide additional assistance to students. In fact, the sale would have entailed additional administrative costs without generating any real savings. Although switching to fixed rates for new student loans after June 2006 mitigated this effect, the payments to guaranteed lenders still depend on short-term interest rates and will continue to be mis-valued.

3.3 Budget Cost Decomposition

The Credit Supplement to the Budget, prepared by OMB, provides a breakdown of subsidy cost across four cost components (defaults, interest, fees and other) for four loan categories (Stafford Subsidized, Stafford Unsubsidized, PLUS and Consolidated). Table 2 reproduces this data from the 2007 Credit Supplement in terms of subsidy rates. The subsidy rate is the present value of net losses divided by the underlying loan principal at origination.

Both programs report similar, and small, subsidy cost component for defaults. As discussed below, this component of cost appears inexplicably low for both programs. For the remaining components, the breakdown of subsidy costs is markedly different across the two programs.

First, the direct program reports large interest income, whereas the guaranteed program reports large interest costs. In the direct program, the government reports interest income as the present value of any interest paid by borrowers in excess of the cost of financing, which is taken to be a Treasury rate. Because the borrower interest rate exceeds the Treasury rate, this item reduces the subsidy cost. In contrast, the interest component in the guaranteed program represents the present value of the net payments paid to private guaranteed lenders (the difference between the lenders rate, which is indexed to the 3-month commercial paper rate, and the borrowers rate), which is positive. Although classified as interest, these payments are more accurately described as covering administrative costs, since the borrower rate typically exceeds lenders’ cost of funds. Administrative costs in the direct program that are borne directly by the federal government, however, are excluded from subsidy estimates. Direct lending administrative costs that entail payments to third parties for tasks such as collecting on loans appear in the category “other.”

Fees levied on borrowers, guaranteed lenders and guaranty agencies reduce subsidy costs. These fees include the upfront application fee on Stafford and PLUS loans in both programs, as well as the 1.05 percent per annum consolidation fee paid by private lenders to the federal government in the guaranteed program. The remaining category, other, largely includes the subsidy cost contribution associated with collecting loans and payments to third parties for performing administrative tasks.

The last two columns of Table 2 show cumulative lifetime default rates and recovery rates. Notice that default rates are for the most part similar in the two programs, reflecting the similar borrower populations. The exception is for consolidation loans, which experience much higher default rates in the direct program. As noted earlier, the
higher default rate can be explained by the reluctance of guaranteed lenders to consolidate loans on the brink of default, but the direct program accepts those loans for consolidation.

The surprisingly high recovery rates seem to arise from two idiosyncrasies in budget reporting: the recovery amounts are not discounted and not all collection costs are deducted. As shown in the next section, adjusting for these factors yields recovery rates in line with experience in the private student loan market.

4. Inferences from the Private Loan Market

Limits on federal borrowing and increasing educational expenses have contributed to the development and rapid growth of a competitive private student loan market. The market primarily serves students who have exceeded federal lending limits, which currently are set at a cumulative amount of $23,000 for undergraduates and a $65,500 combined limit for undergraduate and graduate.

The main players in the private loan market are the largest guaranteed lenders -- Sallie Mae and major national and regional commercial banks. Economies of scale in marketing, systems administration, and funding, and the experience gained from guaranteed lending, give these institutions a competitive advantage over other potential entrants. Although students can obtain private loans on their own, as with guaranteed lending students often rely on the financial aid office for recommendations, which tends to limit direct competition between lenders.

4.2 Adjusting for Federal - Private Differences

Although the private loan market provides data that is useful in estimating the market value of government loans and loan guarantees, various differences make direct cost comparisons problematic. Here we describe the main differences between private and government-backed loans, and propose adjustments to account for the resulting cost differentials.

Borrower population. The federal programs serve a much broader population of students than do private lenders. Since private loans appeal to students who have hit federal borrowing limits, they select for students at high-cost undergraduate institutions and professional students in medicine, law and business. Several factors suggest that federal borrowers are likely poorer credit risks. Federal borrowers eligibility does not depend on a credit score, whereas private lenders use credit scores to discriminate between borrowers, refusing credit entirely below some cutoff. Private lenders also can avoid originating loans at schools where graduates’ employment prospects are weak. On the

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5 There are also various annual limits on federal borrowing. For independent students and those whose parents were denied a PLUS loan the cumulative limits are $46,000 and $138,500, respectively. Some medical school students may be able to borrow up to $40,500 a year (up from $38,500) and $189,125 total.
other hand, private lenders extend credit to students who already have high federal loan balances and who start work with much higher levels of total indebtedness. Comparison of federal default rates with data from private loan providers (drawn from securitization prospectuses) suggests that recent default rates are similar or perhaps slightly lower on private loans. We estimate expected default rates from extensive data on federal loans in the next section, and assume slightly lower private default rates in the base case analysis.

**Loan terms and fees.** Apart from higher rates, private loan terms are less favorable than on Stafford loans along a number of dimensions. Private loans lack the valuable consolidation option, repayment options are more limited, and lenders may be less generous with forbearance. There are no grace or deferment periods, and unlike federal loans, death or disability does not trigger forgiveness. Private loans do offer longer original maturities of up to 20 years, and the mechanisms to collect on defaulted loans are weaker (see section 5.1.2). As on guaranteed loans (but not direct loans), lenders often offer incentives for on-time and electronic payments, etc. Among these non-rate differences, the consolidation option is likely to be the biggest advantage of the federal programs. In Lucas and Moore (2006), we estimate that in every year since 2001 the consolidation option has added more than 2 percent to the market value subsidy rate on new loans. (With the switch to fixed rates, the consolidation option will have less value going forward, but the prepayment option will have more value.)

Competitive pressures have reduced or eliminated origination fees on private loans, hence administrative costs are covered by higher rate spreads. Similarly, most guaranteed lenders pay the federal origination fee on behalf of borrowers. On direct loans, however, borrowers still are required to pay 1.5 percent up front, and the entire fee if they fail to make a timely first payment.

**Administrative costs.** The task of identifying administrative costs and allocating them across activities is complicated by a lack of either government or private data. Common administrative functions include origination, servicing, collection, and general overhead.

Origination costs are likely somewhat higher on private loans because of fees paid to obtain credit scores (including those paid for students who ultimately borrow elsewhere or don’t qualify). Private loans also involve higher contracting costs (e.g., legal expenses) than do direct loans. Loan servicing is a competitive industry, and it is safe to assume that servicing costs are similar for all lenders. Loan collection services can also be obtained at competitive prices, although guarantee agencies are paid a statutory amount that appears to exceed their cost of providing services. We assume similar collection costs for private and direct loans, and adjust for the subsidy component of payments to guarantee agencies in the guaranteed program.

Additional administrative cost arises for guaranteed and private loans because of higher service levels and marketing expenses. Such expenses include information systems, marketing personnel, phone center staff, conference sponsorship, travel to schools, etc.
Private and guaranteed lender financial statements provide some data on administrative costs. Non-interest expense, broken into various categories, is reported on an annual basis. Some costs, such as servicing, apply to portion of the outstanding loan portfolio in repayment, while other costs are incurred for origination activity, but financial reports do not allocate costs by activity. Data from one lender for 2006 is used to make some rough imputations. We attribute 80 percent of personnel, consulting, and occupancy expenses to origination, 100 percent of promotional expenses to origination, and 50 percent of computer and other expenses to origination. Total origination expenses, divided by total volume of Stafford and private originations, is .95 percent. Representing this as an annual rate spread on a 10-year amortizing loan implies an origination cost of 22 bps. The remaining non-interest expenses, divided by the portfolio of loans in repayment, gives an annual cost of 45 basis points. Collection costs are netted against loan recoveries, and not reported separately. We assume these costs add another 5 bps annually, for a total administrative cost rate of 72 bps per annum.

Administrative costs per dollar of loans outstanding in the direct program appear to be lower than per dollar costs for private loans. An annual appropriation to the Department of Education covers the administrative costs of the direct program, although some of these costs are attributable to administering both the direct and guaranteed programs. This appropriation was approximately $800 million in 2006. At that time, the outstanding direct program portfolio was approximately $100 billion and the guaranteed portfolio approximately $300 billion. The Education Department does not publish a breakdown of the appropriation spending across the two programs. Assuming the administrative charge is a stable proportion of loan amounts outstanding over time and assuming a pro rata allocation of the appropriation between the two program would imply a 20 basis point annual administrative charge. At the other extreme, if the entire appropriation supported the direct program, that would imply an 80 basis point annual administrative charge. In our base case analysis of program costs, we have assumed the mid-point of 50 basis points, or $500 million of the $800 million dollar appropriation is spent on the direct program each year. The residual $300 million adds to the 10 basis points per annum to the administrative cost of the guaranteed program. Note that these charges do not include all federal expenses associated with loan recovery.

Cost of capital. We use the interest rate charged by private lenders as the starting point for identifying the market cost of capital for student loans. The difference between the rate charged to students and the one-year Treasury rate includes a risk premium compensating for the systematic risk in loan losses, and also compensation for administrative costs, expected losses, taxes, and any liquidity premium. A broad measure

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6 Simply dividing total non-interest expense over the loan portfolio would be misleading because a large portion of the total is current originations.

7 In our estimates for the federal programs we assign collections costs to the recovered cash flows of loans in default.

8 Unlike the private lender, we assume the federal program is closer to a steady state, so dividing total costs by total loans is a reasonable approximation of annual costs.
of the market cost of capital includes all of these components, since that is what the government would have to pay private lenders to provide this service to students. From a narrower government production cost perspective, it is useful to identify the contribution of these various components, so that the market risk premium can be used to infer a discount rate to apply to expected cash flows.

Private lenders charge students floating rates ranging from LIBOR+2% to LIBOR+7%. The rate offered varies by credit score and educational institution, but LIBOR+4% is typical. Assuming a 30 bp spread between 1-year LIBOR and 1-year Treasury, the interest rate spread over Treasury is 4.3 percent.

A conservative estimate of the risk premium is identified by starting with this 4.3 percent spread, and subtracting estimates of the other spread components. As discussed above, administrative costs are on order of 72 bps. This leaves 3.58 percent for a risk premium, expected default losses, taxes, and a residual liquidity premium. Little evidence is available on these components for consumer credit, but many studies attempt to decompose the credit spread on investment grade corporate bonds (add citations). Different studies come to distinctly different conclusions about the relative importance of risk, taxes and liquidity, but most agree that the fraction of the spread attributable to credit risk increases with bond risk, and is small for high-grade bonds. For investment grade bonds, less than half of the spread is usually attributed to credit risk, which includes expected defaults and a risk premium. These studies use secondary market data, which like the 3.58 percent is net of administrative expenses. Since the default rate on student loans places them in a below-investment-grade category, attributing 2 of the 3.58 percent to credit risk is in line with these decompositions. As described in the next section, we estimate the annual default loss rate from federal student loan data to be 1 percent per annum. Default losses on private loans are similar. This leaves 1 percent attributable to a market risk premium, an estimate we use in the cost analysis in the next section. In our sensitivity analysis, we consider higher levels of the risk premium, which can be interpreted as accounting for a broader measure of opportunity cost.

An alternative approach would be to approximate private lenders’ cost of capital by looking at the weighted average cost of debt and equity capital for public firms in this business. This turns out to be impractical because there are few publicly traded companies whose primary business is making and self-funding private student loans. Further, the few public companies that specialize in private loans have a short history. They also tend to repackage the risk, for instance through securitization structures. Hence the cost of capital for private student loans cannot be accurately inferred from traded debt and equity returns.

5. Estimating Federal Program Costs

The main cost analysis involves projecting the distribution of future cash flows to and from the government over the life of a loan or guarantee obligation, and discounting at risk-adjusted rates. We start by modeling the cash flows associated with the underlying loans, taking into account program rules, borrower behavior and the various options
affecting payment patterns. These cash flows, in combination with rules for payments between guaranteed lenders and the government, also determine the cash flows associated with guaranteed loans.

A sub-sample of student records from the Department of Education’s National Student Loan Database System (NSLDS) provides information on historical borrower payment patterns, which is used to parameterize the model. In particular, we derive new estimates of default and recovery, which are critical inputs into the subsidy rate. We use a sample from the database drawn in January 2006, which contained historical information on loans and borrowers dating back to 1980, although we used the older data only where absolutely necessary. The sample comprises over 10 million loan records and 1 million borrowers. We use Monte Carlo simulation to project future cash flows that depend on stochastic interest rates and borrower behavior. Discounting projected cash flows at the risk-adjusted rates (derived as described above) yields cost estimates for both programs. In addition, we present alternative estimates based on a simple comparison of private and government student loan rates.

5.1 Cash flows

On direct loans, a net outflow of principal occurs when the borrower takes a new loan (less the 1.5% origination fee paid by the borrower). Subsequently, net inflows of repaid principal and outstanding interest flow to the government over time including amounts recovered from default less any recovery costs. The government also incurs ongoing administrative cost, which we apportion to individual loans on a per annum basis.

In the guaranteed program, government cash flows include net transfers to and from lenders (indirectly via guaranty agencies) on each outstanding loan, equal to the difference between the borrower’s interest payment (if any) and the 3-month commercial paper rate plus a spread. This is referred as a Special Allowance Payment, or SAP. Currently the spread is equal to 1.74% per annum for Stafford loans when the borrower is in school, 2.34% for Stafford loans when the borrower is in repayment and 2.64% (less the 1.05% per annum lender consolidation fee) for consolidation loans. The government also makes guarantee payments to lenders for claims on defaulted loans; and pays “retention” fees to guaranty agencies in proportion to their recoveries on defaulted loans.

5.1.1 Effective Maturity and Repayment Status

The time to repayment on federally-backed loans varies from less than a year to over 30 years. Borrowers may prepay their federal loans without penalty, which can significantly shorten loan terms. For example, approximately 8% of originated loans close in less than 5 years, and approximately 60% within 15 years. The future distribution of loan lifetimes may be more drawn out than indicated by historical data, since closure rates at long horizons are based on loans taken out when the federal loan program offered less favorable terms to borrowers.

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9 These estimates treat loan consolidations as an extension of the original loan rather than a new loan. Stafford loan lifetimes would otherwise appear to be much shorter than this.
While in school and for a few months after graduating, borrowers do not need to make payments on federally backed student loans. During this grace period, the federal government pays the interest for subsidized loans, whereas interest accrues on unsubsidized loans. Periods of grace necessarily raise the market-based subsidy cost even for unsubsidized loans, since the interest rate is typically lower than a private lender would charge. Over 95% of loans by originated value are in an in-school or grace period in the year of origination, but less than 10% of loans were in a grace period four years after origination. The average time in-school is approximately 2.5 years (excluding time in loan deferral for subsequent schooling discussed next).

Borrowers are entitled to lengthy payment deferral in times of financial hardship or, for Stafford borrowers, to pursue further studies. Stafford loans are also forgiven in the event of death or disability of the borrower (parent loans are forgiven upon the death, but not disability, of the student). An effect of these provisions is that they may lower reported default rates. Periods of in-school deferment last as long as the borrower remains in school, whereas a borrowers experiencing financial hardship may elect a 3-year payment deferment or payment forbearance period (the former is available only under more restrictive conditions). Analysis of loans in the NSLDS suggests that borrowers enter deferment or forbearance at a rate of approximately 6% per annum for a typical term of 3 years.

The effect of these repayment options is shown in Figure 3, which shows the breakdown of outstanding loan principal by loan status in January 2006 for both the direct and guaranteed program. Overall, only about half of the loans are in repayment, while grace, deferral, forbearance and default account for the remainder.

Borrowers have standard options to extend Stafford loans beyond the basic 10-year maturity even without consolidation. Stafford borrowers with a balance of $30,000 or more from a single lender (whether a single guaranteed lender, or a loan from the direct program) may choose an extended repayment plan of up to 25 years. Income contingent and graduated repayment plans are also available. The right to consolidate Stafford loans also allows borrowers to extend the term of their original loans, as well as to convert floating rate loans to a fixed rate. For some borrowers, consolidation allows them to extend for up to 30 years. Eligibility for term extension depends on the size of the consolidated loan as shown in Table 3.

The OMB treats consolidation loans as new loans rather than the extension of existing loans. This leads to a higher reported loan volume, but a lower subsidy cost per loan than the subsidy estimates reported in this paper, which treat consolidation as an extension of existing loans. Treating a consolidation loan as an extension of an original loan makes it easier to interpret default and recovery experience. This approach also ensures the subsidy cost includes the value of the option to consolidate.

5.1.2 Default and Recovery
Borrower default is an ongoing concern in both the direct and guaranteed lending programs, despite the strong loan enforcement mechanisms that the government has at its disposal. Before direct lending, the guaranteed lending program reported very high default rates. In response, Congress made a number of changes to the Higher Education Act. Chief among them was the use of cohort default rates as a performance measure and as a criterion for schools to retain access to federal student loans and grant funding. Since the adoption of these measures, new default claims in both the direct and guaranteed lending programs have more than halved.

The strength of the US economy and the increased use of deferment, forbearance and consolidation have also contributed to lowering default rates. Offering more generous terms to students is costly, however, since it may delay an inevitable default and make recovery more difficult. Table 4 below reports default claims as a percent of outstanding balance for 1990, 1996 and 2005.

Figure 4 shows the average default rate of loans issued between 1996 and 2006 by years since entering repayment for guaranteed Stafford, guaranteed consolidated, direct Stafford and direct consolidated. Average default rates are around 2 percent per annum. Stafford loans experience higher levels shortly after entering repayment, which may in part reflect the cumulative effect of in-school grace periods (since a borrower cannot default while he or she remains in school even though adverse circumstances may arise that impair a borrower’s current and future ability to repay their loans). Consolidated direct loans report strongly higher default rates than consolidated Stafford loans because the Education Department frequently consolidates borrowers that guaranteed lenders consider too risky. Even though guaranteed lenders bear virtually no credit risk, the administrative expense of consolidating a borrower’s loans and resolving default is a sufficient disincentive to consolidate risky borrowers. Data confirms that borrowers that consolidate defaulted loans are more likely to default on their consolidation loans than other borrowers. We attribute the cost of these defaults to the program in which the original loans were originated rather than the program that consolidated them.

OMB reports recovery rates on student loans that far exceed those on other forms of unsecured consumer credit, but as discussed in Section 3.3 their measure neglects collection costs and time value. Relying instead on NSLDS data, we find that individual loans exhibit significant variability in recoveries, with some defaulted loans resolved quickly and others remaining uncollected for more than 10 years. The typical pattern suggests that collection rates diminish over time. Applying a risk adjusted discount rate (equal to the average interest rate over the data period plus our assumed 1% risk premium) and subtracting a 16% recovery cost suggests a recovery rate of around 50% of the defaulted principal. Combining this with the annual default rate of 2% per annum implies losses from default equal to 1% of principal outstanding per annum.

5.2 Simulating Cash Flows

\footnote{Student loans are not dismissed in bankruptcy. The government can collect through the Treasury Offset Program.}
Cash flows for both programs depend on the stochastic path of future interest rates, program rules, and borrower behavior. These are modeled using Monte Carlo simulation. Each month a random draw from a normal distribution determines the innovation in the short-term interest rate, and the corresponding term structure is derived from the Cox Ingersoll Ross (CIR) model (see Appendix 2 for a complete description of the interest rate model and the parameters used in estimation). Variation in interest rates affects the discount rate and guaranteed lender payments.

Monthly loan repayment cash flows depend on various borrower behaviors: whether the student is in-school; the borrowers repayment plan; consolidation; default, recovery, prepayment; and an administrative charge. Appendix 2 contains a description of how we simulate the cash flows that depend on stochastic borrower behavior. It also describes the aggregation of cash flows across representative loan groupings.

The cash flow model is calibrated under the following base case assumptions:

**Borrower Interest Rates:** From June 2006 onwards, borrowers will pay a fixed rate of 6.8% p.a. on all new Stafford loans. When those loans are subsequently consolidated, the interest charged on that portion of the consolidation loan that comes from post 2006 Stafford loans will be at a 6.8% rate (plus up to 0.075% after rounding).

**Repayment Horizons:** A typical loan repays over a 20 year term, but any individual loan can be repaid over shorter or longer horizons. The probability of longer repayment is positively correlated with the borrower’s balance. For borrowers entering repayment, approximately one-third of all loan value is in each of three balance categories, and respectively 15%, 40% and 60% of borrowers in each category take up the maximum extension options.

**Default Losses:** The value of default losses each year is equal to 1% of outstanding balances in the direct program and 1.1% in the guaranteed program. The guaranteed program losses are assumed to be higher because the federal government pays more to Guaranty Agencies for their collections from defaulted borrowers than they do to private contactors in the Direct Program. This estimate may be refined in future drafts.

**Non-Collection Related Federal Administrative Expenses:** The federal government incurs direct administrative expenses for both programs. These costs are not included in official budget subsidy estimates, but they are included in the more comprehensive estimates here. Each year, we assume the department directly spends 0.5% of outstanding principal administering the direct program and 0.1% administering the guaranteed program. The administrative costs borne by guaranteed lenders in the guaranteed program do not directly affect subsidy rates.

**Guaranteed Lender Payments:** The federal government pays guaranteed lenders a spread above the quarterly reset 3-month commercial paper rate. That spread varies with the type of loan and its payment status as described earlier, and terminates upon default.
**Loan Origination Fee Receipts:** The government charges borrowers a 3% origination fee in both programs, which reduces the subsidy cost by a corresponding amount. In the guaranteed program, guaranteed lenders often pay this fee for the borrower. In the direct program, the Department of Education charges half of the 3% fee up-front and only levies the remaining 1.5% if borrowers fail to enter repayment on time. For simplicity, we assume one-half of borrowers enter repayment on time, reducing the total fee (in present value) to 2.25%.

**Adjustments for Liquidity:** The average spread of 30 basis points between secondary market Treasury bill rates and rates on the safest commercial paper suggests a significant funding advantage to the federal government. As discussed in the conclusion, there are several possible explanations for the difference. A broad measure of the opportunity cost of providing federal credit would include some elements of the spread as a cost of the program. In this draft in our base case, we treat the commercial paper spread as a financing cost of the direct and guaranteed programs, borne by the federal government in the former and lenders in the latter.

**Adjustments for Federal Revenue Effects:** The companies that serve the direct and guaranteed programs pay federal corporate income taxes. Ideally, the corporate income taxes paid should be taken into account in calculating the net federal outlay. However, current budget practice does not recognize income tax receipts in subsidy estimates. A recent study by Price Waterhouse Coopers (2005) estimated that the guaranteed lending program generates corporate income tax with a present value of 1.5 cents per dollar of loans originated, which translates to an approximate per annum tax receipt of 20 basis points per dollar outstanding. The direct program also generates corporate income taxes from IT, servicing and collections contracts with private companies. We assume this generates no more than 5 basis points of tax revenue leaving a 15 basis point per annum tax differential between the direct and guaranteed programs. In our base case subsidy estimates we ignore this differential, but we do account for the difference in the sensitivity analysis.

### 5.3 Risk Adjusting Discount Rates

A major goal of this analysis is to understand the effect of market risk on estimated program costs. As described in Section 4.2, an analysis of the private student loan market suggests a conservative estimate of the credit risk premium for student loans is 1%. A more comprehensive measure of market capital costs suggests a spread as high as 2.8%. The 1% credit risk premium is used in the base case estimates, but the sensitivity analysis reports results for risk premia, ranging from 0 to 3 percent.

The cost of capital is also affected by the term premium: the difference between long term and short term Treasury rates. The valuation model incorporates the term premium in the interest rate model, but for simplicity we treat the risk that generates the term premium as independent from the risk driving credit spreads.
To the extent that program cash flows are proportional to loan cash flows, one can simply apply the risk-adjusted discount rate for student loans to other program cash flows. The risk-adjusted discount rate is a maturity-matched interest rate from the interest rate model plus a credit risk premium. Applying this discount rate to all cash flows is a reasonable approximation for the direct program, assuming that administrative costs are proportional to loan cash flows.

Correctly discounting risky cash flows in the guaranteed program is more complex than in the direct program, because guaranteed cash flows have different exposures to market risk. With a 100% credit guarantee, the federal government’s cash flows are equivalent to directly lending to the student but financing the loan by borrowing from the private sector instead of the Treasury, and contracting for origination, servicing, and part of collection (we call these equivalent credit arrangements implied loans). The cash flows from the implied student loan have interest rate risk, since they are made at a fixed rate that is unrelated to market interest rates, and credit risk. They are discounted at the same rate as cash flows in the direct program. In contrast the implied loan made by guaranteed lenders to the federal government is largely unaffected by default risk. Specifically the guaranteed lender is assured of receiving full repayment of principal and interest, so there is a component that is risk-free. At the same time, there is the risk that default, prepayment or consolidation will terminate or reduce the stream of lender payments, which introduces an element of market risk.

To incorporate the effect of these risks on the value of direct and guaranteed loans, we overlay a simple two state model of default on top of our interest rate model to provide state dependent discount rates (or state prices). Each state of the model corresponds to an interest rate and a borrower default state (i.e. whether default has occurred or not) allowing us to specify cash flows in each of those states and discount them accordingly. The appropriate discount rates differential between default and non-default states is inferred from the spread between risky and risk free loans (and justified by a no-arbitrage argument). Appendix 2 explains the model in detail.

5.4 Base Case Subsidy Estimates

Table 5 presents subsidy estimates for newly originated loans in academic year 2006 (July 1, 2006 to June 30, 2007) under the base case assumptions outlined above. The overall subsidy estimate for each program is computed by averaging over representative groupings of loans by subsidized status and maximum available repayment horizon. The difference between subsidized and unsubsidized loans is the present value of in-grace and in-deferment interest paid by borrowers with unsubsidized loans that is not paid by borrowers with subsidized loans. Considering a typical subsidized borrower will spend around 3 years in grace and deferment, the 6.8% per annum foregone interest adds up to a subsidy of about 12% of the loan amount.

\[ \text{11 To simplify the discussion, we are ignoring any borrower incentives offered by lenders, retention allowances to guaranty agencies and the various one-time fees between lenders and the government.} \]
Subsidy rates vary with the term of loan repayment. Allowing borrowers to extend a 10-year Stafford year loan to 20-years raises the subsidy cost for that loan by around $3 per hundred dollars originated. This increase would be even higher, but many borrowers fail to take advantage of term extension options and frequently pay their loans off early. The current version does not incorporate a behavioral response of prepayment to interest rate conditions, which would further increase government cost.

Guaranteed loans have a consistently higher subsidy rate than do direct loans. In this draft, subsidies for both programs are computed under the assumption that actual administrative and capital costs are similar across both programs, but the net income payments to guaranteed lenders and guaranty agency collection fees are significantly more than required to cover those costs in the guaranteed program.

Looking to the future, subsidy rates for new loans may be considerably different from the estimates for 2006 reported in Table 5. The most obvious cause of future variation in new loan subsidy rates is changes in interest rate conditions. This is because borrower interest rates are fixed at 6.8% per annum for all new Stafford loans, whereas the government’s opportunity cost moves with prevailing interest rates. Thus, if interest rates go up next year, subsidy rates will rise, whereas if interest rates decline so will subsidy rates. Figure 5 shows average, 10th and 90th percentiles of subsidy estimates for each of the next 10 years. To make these forecasts, we use the interest rate model combined with current yield curve information to provide simulated paths of future interest rates to determine starting conditions for each year. We assume loan cash flow performance is consistent with the assumptions of the base case (but appropriate to interest rate conditions). As the horizon lengthens, the course of future interest rates becomes more uncertain so the band of subsidy values widens in both programs. The range of subsidy values is extremely wide, and would be even wider were the analysis to account for the impact on lending volumes of these changes in interest rates, which we abstract from in this draft.

5.5 Sensitivity Analysis

Aggregate subsidy estimates under alternative assumptions are shown in Table 6. Subsidy estimates are quite sensitive to the assumed risk premium. A one percent higher (lower) risk premium than in the base case raises (lowers) subsidy rates by 7 dollars per hundred. For the direct program, this is most easily understood as the higher discount rate reducing the value of future repayments. On the guaranteed loans, the effect of market risk is to raise the present value of guarantee payments made on defaulted loans. The credit risk premium has a small effect on the present value of net income payments to guaranteed lenders, as discussed in section 5.3. The effective duration of the loans also affects value, with loan extension generally increasing cost. Table 6 also reports subsidy costs with 25% faster and slower loan repayment rates, which serves to lengthen or shorten the average loan term by approximately 4 years. The increase (decrease) raises (lowers) subsidy costs by about 2 dollars per 100.

To compare the cost of the two programs, it is useful to break subsidy costs into their component parts. Table 7 reports a breakdown into market risk, default losses, up-front
fees and other administrative expenses. The cost attributable to each component is found by sequentially removing each cost element from cash flows or discount rates, computing the new subsidy cost and then reporting the difference. Any residual subsidy cost is driven by the difference between the student interest rate and risk free rate of return. Market risk and default losses are the most important elements of the direct loan subsidy. The guaranteed lender payments add significantly to the guaranteed program subsidy cost.

Finally, Table 8 reports subsidy costs for the direct and guaranteed programs for a variety of policy alternatives. One option is to lower the guaranteed lender payments to bring the guaranteed subsidy closer to the direct loan subsidy rate. The first two rows of Table 7 report the predicted subsidy estimates after lowering lenders payments by 0.5% and 1.0% per annum, respectively. The effect of the 1.0% reduction is to bring the subsidy in the guaranteed program to within 3% of the direct program. Another set of alternatives relate to the interest rate paid by borrowers. Switching from variable to fixed interest rates on Stafford loans has increased the subsidy cost for 2006 by approximately 2 dollars per hundred, in part because fixed rate loans should have a premium above variable rate loans and in part because our long term interest rate projections implies the variable rate loan will be above 6.8%. If market interest rates continue to increase, subsidy costs on loans originated after 2006 could be significantly higher than they would be under the variable rate policy. Several proposals to lower borrower rates are being discussed by members of Congress. One proposal would cut the borrower rate in half over the entire life of the loan, which would increase the subsidy cost by approximately 15 dollars per hundred. (In fact, the subsidy cost could increase by even more than this if the lower rates cause borrowers to lower their rate of prepayment and switch to longer term repayment plans).

5.6 Alternative Estimates

A ballpark market value estimate of the cost to the government of direct lending is obtained by comparing the rate charged to students on private loans with the rate charged by the government, and discounting the annual savings. Although this does not control for the differences between private and direct loans and greatly oversimplifies the pattern of cash flows, it provides a useful comparison point, and confirms that the low subsidy cost for direct loans in the budget is probably well below the true cost to taxpayers.

Like the earlier calculation of the cost of capital, the estimate is based on typical terms for floating rate Stafford loans and private loans made in recent years, where LIBOR+4% approximates the average rate. Stafford loans carry a rate based on a 3-month Treasury rate that resets annually, plus a spread. The spread equals 1.7% when the student is in school, grace or deferment, and 2.3% otherwise. Approximating the difference between LIBOR and 3-month Treasury at 30 bps, and assuming a 2 percent average spread on federal loans, students typically pay 2.3% more per annum on private loans than on federal loans. We assume further that both types of loans amortize over 10 years.

12 The order that different cost elements are removed has a modest impact on the cost attribution to different components.
abstract from the effects of prepayment and default, set LIBOR to 5%, and discount at LIBOR + 4%. This yields an estimated present value interest savings to students of $10,463 per $100,000 borrowed, or a 9.8% subsidy rate. The higher subsidy rate in the more complete analysis can be attributed to the value of the various extension and deferral options.

6. Discussion and Conclusions

We conclude with further discussion of two important issues. First, what is the government’s cost advantage over the private sector in funding fully guaranteed student loans? Second, why has the guaranteed program continued to be funded despite its much higher budget cost than direct lending?

Guaranteed lenders routinely obtain funding by securitizing parcels of previously originated federal loans and selling these asset-backed securities to investors, at a weighted average rate slightly over LIBOR. This suggests that private investors do not view guaranteed loans as perfect substitutes for Treasury securities, despite the 98% to 100% credit guarantee.\(^\text{13}\) In addition, lenders bear underwriting, SEC filing, and other administrative fees that add to the total cost of capital. In comparison with the cost of Treasury funding for direct loans, it appears that guaranteed lenders pay 25-35 basis points more to borrow. What accounts for the higher cost? One factor is that a guaranteed loan is not truly risk-free -- lenders who fail to administer loans according to ED policy and regulations may have the guarantee voided for those loans. The exemption of Treasury interest from state and local taxes also lowers Treasury rates relative to LIBOR. Further, securitized student loans are less liquid than Treasury securities. The prepayment and extension options add uncertainty about maturity, also increasing funding cost (as evidenced by higher spreads on the tranches of securitizations that absorb the maturity risk). However, these options also increase the cost of funding direct loans, and like the risk premium should be included in a market value cost estimate for either program. Taking into account the government’s funding cost advantage would result in a lower estimated market value cost of direct loans than reported here.

Prior to the mid-1970s, individual federal agencies raised funds separately, but recognition of the cost advantage to centralized borrowing led to a policy of consolidating federal borrowing through the Treasury, via the Federal Financing Bank. Since that time, growth in federally guaranteed loan programs has sacrificed some of this advantage. There may be instances when private intermediation adds value, for instance through better screening or monitoring of borrowers. However in the case of student loans, which have categorical entitlement and an almost full credit guarantee, it is not clear that much value is added by private intermediation. Beyond providing loans, the guaranteed program channels money to students, schools, and guarantee agencies, but such subsidies presumably could be better targeted and controlled if they were separated from the lending function.

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\(^\text{13}\) This discussion is based on securitizations of floating rate loans, and prospectus data from Sallie Mae on recent issues.
As discussed earlier, the Federal Credit Reform Act of 1990 made the direct student loan program feasible from a budgetary perspective. Since then the political fortunes of the two programs have shifted, sometimes abruptly, with changes in control of Congress and the White House. The non-partisan New America Foundation provides a lively account of the political history of the student loan program. They attribute an analysis by the Bush administration indicating that direct loans would be less costly and simpler to administer as prompting Congress to create a direct lending pilot program in 1992. In 1993, the newly elected President Clinton proposed saving money by replacing the guarantee program with direct loans. As part of the 1993 budget agreement, Congress voted to phase in direct lending. The plan started with colleges that volunteered, but allowed the Secretary of Education to require colleges to switch until at least 60 percent of loans nationwide were direct. In 1994, the new Republican leadership in Congress targeted direct lending for elimination, presumably due to pressures from the various constituencies that benefit under the guaranteed program. However, many schools were dissatisfied with the guaranteed loan system, and hundreds of institutions were already participating in the direct loan program. Ultimately, Congress stopped short of eliminating direct lending. Instead, they passed a law that prohibited the Department of Education from encouraging or requiring colleges to switch to the direct loan program. Recent proposals by the new democratic majority in Congress would use savings from inducing schools to switch to the direct program to fund more favorable loan terms for students.

This history suggests that budget cost is a factor, but not always a decisive one, in influencing the structure of the student loan programs. The market value estimates developed here indicate that the cost to taxpayers of both programs is significantly understated in the budget. Perhaps greater recognition of the high economic cost of both programs will make efficiency a higher priority in the future.
Table 1: Federal Student Loans Outstanding, 1998-2005

<table>
<thead>
<tr>
<th></th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
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<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
</tr>
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<tbody>
<tr>
<td><strong>FFEL</strong></td>
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<td></td>
</tr>
<tr>
<td>Unconsolidated</td>
<td>74,727</td>
<td>92,760</td>
<td>106,220</td>
<td>122,423</td>
<td>129,757</td>
<td>130,455</td>
<td>142,405</td>
<td>148,391</td>
</tr>
<tr>
<td>(Stafford &amp;</td>
<td></td>
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<td>PLUS)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consolidated</td>
<td>9,675</td>
<td>20,008</td>
<td>27,891</td>
<td>32,384</td>
<td>49,434</td>
<td>79,017</td>
<td>100,176</td>
<td>138,457</td>
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<td>Subtotal</td>
<td>84,402</td>
<td>112,768</td>
<td>134,111</td>
<td>154,807</td>
<td>179,191</td>
<td>209,472</td>
<td>242,581</td>
<td>286,848</td>
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<td><strong>Direct</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconsolidated</td>
<td>26,937</td>
<td>33,763</td>
<td>43,091</td>
<td>47,958</td>
<td>50,264</td>
<td>51,013</td>
<td>52,090</td>
<td>47,679</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consolidated</td>
<td>4,733</td>
<td>12,067</td>
<td>14,622</td>
<td>22,526</td>
<td>29,807</td>
<td>33,507</td>
<td>37,155</td>
<td>47,027</td>
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<tr>
<td>Subtotal</td>
<td>31,670</td>
<td>45,830</td>
<td>57,713</td>
<td>70,484</td>
<td>80,071</td>
<td>84,520</td>
<td>89,245</td>
<td>94,706</td>
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<tr>
<td><strong>Total</strong></td>
<td>116,072</td>
<td>158,598</td>
<td>191,824</td>
<td>225,291</td>
<td>259,262</td>
<td>293,992</td>
<td>331,826</td>
<td>381,554</td>
</tr>
</tbody>
</table>

Source: OMB, as reported in the budget appendix.
Table 2: Composition of Subsidy Costs. Source: Federal Credit Supplement 2006

<table>
<thead>
<tr>
<th></th>
<th>Subsidy Rate</th>
<th>Composition of Subsidy</th>
<th>Default Rate</th>
<th>Recovery Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Defaults (net of recovery)</td>
<td>Interest</td>
<td>Fees</td>
</tr>
<tr>
<td>Ford Direct Loan Program</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weighted Average of Total Obligations</td>
<td>2.05</td>
<td>1.31</td>
<td>-2.66</td>
<td>-1.67</td>
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<tr>
<td>Subsidized Stafford</td>
<td>9.83</td>
<td>0.67</td>
<td>6.44</td>
<td>-3.00</td>
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<tr>
<td>Unsubsidized Stafford</td>
<td>-8.28</td>
<td>0.82</td>
<td>-12.59</td>
<td>-3.00</td>
</tr>
<tr>
<td>PLUS</td>
<td>-6.37</td>
<td>0.89</td>
<td>-11.97</td>
<td>-4.00</td>
</tr>
<tr>
<td>Consolidation</td>
<td>4.37</td>
<td>1.92</td>
<td>-0.99</td>
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<tr>
<td>Family Federal Education Loan Program</td>
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<tr>
<td>Weighted Average of Total Obligations</td>
<td>9.87</td>
<td>0.89</td>
<td>11.12</td>
<td>-5.54</td>
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<tr>
<td>Subsidized Stafford</td>
<td>17.78</td>
<td>0.86</td>
<td>17.62</td>
<td>-3.25</td>
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<tr>
<td>Unsubsidized Stafford</td>
<td>1.12</td>
<td>0.96</td>
<td>0.79</td>
<td>-3.25</td>
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<tr>
<td>PLUS</td>
<td>-0.01</td>
<td>0.88</td>
<td>-1.73</td>
<td>-3.25</td>
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<tr>
<td>Consolidated</td>
<td>12.20</td>
<td>0.88</td>
<td>15.46</td>
<td>-8.26</td>
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</table>

Table 3: Allowable Term by Balance

<table>
<thead>
<tr>
<th>Term</th>
<th>Balance Must be at Least</th>
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<tbody>
<tr>
<td>10 yrs</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>$7,500</td>
</tr>
<tr>
<td>15</td>
<td>$10,000</td>
</tr>
<tr>
<td>20</td>
<td>$20,000</td>
</tr>
<tr>
<td>25</td>
<td>$40,000</td>
</tr>
<tr>
<td>30</td>
<td>$60,000</td>
</tr>
</tbody>
</table>

Allowable term for extended and graduated repayment plans in the direct program and for newly consolidated loans in both programs. Balance refers to total balance of loans in the direct program for direct program extensions and total balance of loans consolidated for consolidation term extension. In the guaranteed program, borrowers with balances of more than $30,000 can elect a 25 year extended repayment term on their original loans.
Table 4: Default claims as a percentage of the outstanding federal loan portfolio

<table>
<thead>
<tr>
<th>Budget Year</th>
<th>1990</th>
<th>1996</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outstanding Loan Portfolio ($million)</td>
<td>49,890</td>
<td>57,557</td>
<td>242,581</td>
</tr>
<tr>
<td>Default Claims ($ millions)</td>
<td>2,384</td>
<td>1,428</td>
<td>3,818</td>
</tr>
<tr>
<td>Percentage of Loans in Default</td>
<td>4.8</td>
<td>2.5</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 5: Base Case Market Based Subsidy Estimates for New Stafford Loans Originated in Award Year 2006

<table>
<thead>
<tr>
<th>Unsubsidized Loans</th>
<th>Direct</th>
<th>Guaranteed</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0- 20,000</td>
<td>7.8</td>
<td>16.8</td>
<td>9.0</td>
</tr>
<tr>
<td>$20,000 – 60,000</td>
<td>9.9</td>
<td>20.1</td>
<td>10.2</td>
</tr>
<tr>
<td>$60,000 +</td>
<td>11.6</td>
<td>23.1</td>
<td>11.5</td>
</tr>
<tr>
<td><strong>Weighted Average Subsidy of Unsubsidized Loans</strong></td>
<td><strong>9.7</strong></td>
<td><strong>20.0</strong></td>
<td><strong>10.3</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subsidized Loans</th>
<th>Direct</th>
<th>Guaranteed</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0- 20,000</td>
<td>21.8</td>
<td>29.6</td>
<td>7.8</td>
</tr>
<tr>
<td>$20,000 – 60,000</td>
<td>24.3</td>
<td>33.0</td>
<td>8.7</td>
</tr>
<tr>
<td>$60,000 +</td>
<td>26.6</td>
<td>36.3</td>
<td>9.7</td>
</tr>
<tr>
<td><strong>Weighted Average Subsidy of Subsidized Loans</strong></td>
<td><strong>24.2</strong></td>
<td><strong>33.0</strong></td>
<td><strong>8.8</strong></td>
</tr>
</tbody>
</table>

| Program Average | 17.0 | 26.5 | 9.5 |
Table 6: Parameter Sensitivity of Subsidy Estimates

<table>
<thead>
<tr>
<th></th>
<th>Direct</th>
<th>Guaranteed</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>17.0</td>
<td>26.5</td>
<td>9.5</td>
</tr>
<tr>
<td>Varying Credit Risk and Credit Risk Premium</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High Credit Risk Premium (2% p.a.)</td>
<td>29.7</td>
<td>37.5</td>
<td>7.8</td>
</tr>
<tr>
<td>Low Credit Risk Premium (0.5% p.a.)</td>
<td>13.4</td>
<td>23.4</td>
<td>10.0</td>
</tr>
<tr>
<td>No Credit Risk Premium</td>
<td>9.4</td>
<td>20.0</td>
<td>10.6</td>
</tr>
<tr>
<td>No Default or Credit Risk Premium</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed of Repayment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25% Faster than Base Case</td>
<td>14.4</td>
<td>22.7</td>
<td>8.2</td>
</tr>
<tr>
<td>25% Slower than Base Case</td>
<td>19.3</td>
<td>29.7</td>
<td>10.4</td>
</tr>
<tr>
<td>Sensitive to Interest Rates</td>
<td>Xxx</td>
<td>xxx</td>
<td>xxx</td>
</tr>
<tr>
<td>Other</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longer Stafford Repayments/Reduced</td>
<td>17.2</td>
<td>27.9</td>
<td>10.8</td>
</tr>
</tbody>
</table>

Table 7: Components of Subsidy Cost

<table>
<thead>
<tr>
<th>Base Case Subsidy Cost</th>
<th>Direct</th>
<th>Guaranteed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>17.0</td>
<td>26.5</td>
</tr>
<tr>
<td>Up Front Fees</td>
<td>-2.3</td>
<td>-3</td>
</tr>
<tr>
<td>Federal Non-Collection Administrative Expenses</td>
<td>3.6</td>
<td>0.8</td>
</tr>
<tr>
<td>Lenders spread above CP rates</td>
<td>-</td>
<td>12.8</td>
</tr>
<tr>
<td>Liquidity Charge</td>
<td>2.1</td>
<td>2.1</td>
</tr>
<tr>
<td>Default Losses net of Recovery and Collections Costs</td>
<td>8.1</td>
<td>8.1</td>
</tr>
<tr>
<td>Risk Premium for Credit Risk</td>
<td>9.5</td>
<td>9.5</td>
</tr>
<tr>
<td>Residual: Net Interest</td>
<td>-3.9</td>
<td>-3.9</td>
</tr>
</tbody>
</table>
Table 8: Subsidy Cost of Alternative Policies

<table>
<thead>
<tr>
<th>Program Terms</th>
<th>Direct</th>
<th>Guaranteed</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>17.0</td>
<td>26.5</td>
<td>9.5</td>
</tr>
<tr>
<td>Current CP rules less 0.5%</td>
<td>17.1</td>
<td>23.3</td>
<td>6.2</td>
</tr>
<tr>
<td>Current CP rules less 1.0%</td>
<td>17.0</td>
<td>20.0</td>
<td>2.9</td>
</tr>
<tr>
<td>Floating rates as under 1998-2006 law</td>
<td>14.2</td>
<td>11.0</td>
<td>3.2</td>
</tr>
<tr>
<td>Floating rates but without SAP floor</td>
<td>12.0</td>
<td>20.9</td>
<td>8.9</td>
</tr>
<tr>
<td>3.4% Interest rate on loans - without interest rate response</td>
<td>34.5</td>
<td>43.2</td>
<td>8.8</td>
</tr>
<tr>
<td>3.4% Interest rate on loans - with interest rate response</td>
<td>xxx</td>
<td>xxx</td>
<td>xxx</td>
</tr>
<tr>
<td>90% federal guarantee</td>
<td>17.1</td>
<td>25.0</td>
<td>7.8</td>
</tr>
<tr>
<td>75% federal guarantee</td>
<td>17.2</td>
<td>22.5</td>
<td>4.7</td>
</tr>
</tbody>
</table>
Figure 1:

Percent of New Federal Student Loans Made in the Direct Loan Program

Source: The New America Foundation

Figure 2: Distribution of Loan Lifetimes

Source: Estimates from 2006 sample of the National Student Loan Database.
Figure 3: Status of Direct and Guaranteed Loan Portfolio, January 2006

**Direct Loans**
- Deferred: 11%
- Forbearance: 11%
- Default: 11%
- Grace: 21%
- Repayment: 46%

**Guaranteed Loans**
- Deferred: 11%
- Forbearance: 8%
- Default: 8%
- Grace: 19%
- Repayment: 54%
Figure 4: Default rates by years since entering repayment
Figure 5: Distribution of Future Subsidy Costs Given Interest Rate Uncertainty in the Direct Lending Program
Figure 6: Distribution of Future Subsidy Costs Given Interest Rate Uncertainty in the Guaranteed Lending Program
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Appendix I
Description of NSLDS Data

The Department of Education administers the National Student Loan Database System (NSLDS), a record keeping system that tracks the status of individual loans and borrowers. The Congressional Budget Office receives an annual sub-sample of loan and borrower records each January, which it uses to make cost estimates. The database comprises multiple linked files containing current and historical information about borrowers and their loans. The files used to produce market based subsidy estimates in this paper are:

**Loan File:** The file comprises one record per loan on the type of loan (direct or guaranteed, consolidated or original), the date the loan was taken, the amount disbursed, the principal outstanding at the time the sample was drawn, the current status of the loan and the academic level of the student when the loan was taken. Each loan record also contains a unique identifier for the borrower, school and guaranty agency associated with the loan making aggregation of loans by borrower possible.

**Loan Status History File:** The file contains a sequence of records with dates and codes for each loan’s status changes. A status change occurs for various reasons including: entering repayment, default, deferment, forbearance, consolidation and payment in full. The historical timing of status changes provides a basis for estimating the probability that new loans transition through the various statuses over their lifetime.

**IRS and Guaranty Agency Collections Files:** These files track the timing and amount collected by the IRS, Guaranty agencies, the Department of Education and their contracted agents from borrowers with guaranteed loans in default. No recovery information is available on direct program loans in default. The files contain the amount collected and date of collection for each defaulted loan. This information is combined with historical loan status changes of defaulted loans in the loan status history file to compute a recovery rate on defaulted guaranteed loans, which we assume is very similar to that in the direct program.

Several features limit the usefulness of this data set for estimating loan cash flows over time. Except for the collections on defaulted loans, the CBO sample of NSLDS loans does not contain a record of borrower payments over time. Similarly, when the sample is drawn each January, only the current level of outstanding principal is recorded. Another problem is that repayment plans are not reported making it difficult to infer loan lifetimes and principal down payment.
Appendix 2
Modeling Assumptions (INCOMPLETE)¹⁴

Interest Rates

We adopt the Cox-Ingersoll-Ross (CIR) model to simulate future paths of Treasury rates. In the CIR model, the instantaneous interest rate, \( R(t) \), is the sum of a constant and \( n \) factors, \( z_i(t) \), \( i = 1, \ldots, n \), the state variables in the model:

\[
R(t) = \bar{R} + \sum_{i=1}^{n} z_i(t)
\]

Each factor obeys a mean reverting square root process:

\[
dz_i(t) = \kappa_i \left[ \theta_i - z_i(t) \right] dt + \sigma_i \sqrt{z_i(t)} dZ_i(t)
\]

where \( \theta_i \) is the mean reverting rate, \( \kappa_i \) is the speed of mean reversion, \( \sigma_i \) is the volatility and \( dZ_i(t) \) is a standard Weiner process independent across factors.

Under the risk neutral (or equivalent martingale) measure

\[
dz_i(t) = \bar{\kappa}_i \left[ \bar{\theta}_i - z_i(t) \right] dt + \sigma_i \sqrt{z_i(t)} dZ_i(t)
\]

where

\[
\bar{\kappa}_i = \kappa_i + \lambda_i
\]

and

\[
\bar{\theta}_i = \frac{\kappa_i \theta_i}{\kappa_i + \lambda_i}
\]

\( \lambda_i \) is the constant market price of risk for factor, \( i \). The time \( t \) price of a zero coupon bond with unit coupon and expiring at \( T \) is

\[
p(t, T) = e^{-\bar{\kappa}(T-t)} \prod_{i=1}^{n} A_i(t, T) e^{-B_i(t, T) y_i(t)}
\]

where

\[
A_i(t, T) = \left[ \frac{2 \gamma_i \exp \left[ \left( \gamma_i + \bar{\kappa}_i \right) \left( T - t \right) / 2 \right] - \exp \left[ \gamma_i \left( T - t \right) / 2 \right]}{\left( \gamma_i + \bar{\kappa}_i \right) \left[ \exp \left[ \gamma_i \left( T - t \right) / 2 \right] - 1 \right] + 2 \gamma_i} \right]^{-\frac{2 \bar{\kappa}_i \theta_i}{\sigma_i^2}}
\]

¹⁴ This appendix uses some of the text, figures and equations in appendix 2 of Lucas and Moore (2007).
\[ B_i(t, T) = \frac{2 \exp[\gamma_i (T-t) - 1]}{(\gamma_i + \bar{\kappa}) \left[ \exp[\gamma_i (T-t)] - 1 \right] + 2\gamma_i} \]

and

\[ \gamma_i = \sqrt{\kappa_i^2 + \sigma_i^2} \]

The yield to maturity, \( y \), of a zero-coupon bond maturing at \( T \) is

\[ y(t, T) = \frac{-\ln p(t, T)}{T-t} \]

Jagannathan, Kaplin and Sun (2003) estimate the following factors from the two-factor model using weekly LIBOR rates of various maturities from 1995 through 1999:

<p>| Table B1: Parameters for the Cox-Ingersoll-Ross 2-factor interest rate model |</p>
<table>
<thead>
<tr>
<th>Factor</th>
<th>( \kappa )</th>
<th>( \theta )</th>
<th>( \sigma )</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.392</td>
<td>0.272</td>
<td>0.0153</td>
<td>-0.00038</td>
</tr>
<tr>
<td>2</td>
<td>0.0532</td>
<td>0.0162</td>
<td>0.0430</td>
<td>-0.0592</td>
</tr>
<tr>
<td>( \bar{R} )</td>
<td>-0.229</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Under these parameters, factor 1, with the stronger degree of mean reversion, drives the gap between long and short term rates, whereas factor 2 determines long term rates. We subtract 20 basis points from \( \bar{R} \) to reflect the average spread between 3-month Treasury and LIBOR yields. With these parameters, the Model implies an long run average 3 month Treasury bill rate of XX% and 10 year Treasury bill rate of YY%.

For each Monte Carlo run, initial levels of the states variables are calibrated to fairly price an initial 3-month t-bill and 10 year Treasury bond rate. For each simulation, the instantaneous rate is sampled monthly for as many months as the maximum maturity of the student loan, using a discrete approximation of the risk neutral process in equation . Using the risk-neutral path allows us to apply a time varying monthly discount rate, \( d_i \), to all cash flows for each sample path:

\[ d_i^j = -\ln p\left( \frac{t}{12}, \frac{t+1}{12} \right) \]

Averaging over the discounted value of all sample paths provides the net present value of nominally risk free cash flows.

**Cash Flows**
Loans originate at time 0, begin repayment at time $T^R$, and have a maturity of $T^M$ so the loan is repaid in $T^R + T^M$ months. $T^M$ depends on whether the consolidation option is exercised or, in the counterfactual case, the loan term is extended. The original maturity of Stafford loans is 10 years. Section B.3 of this appendix describes the stochastic rules governing consolidation and extension.

Interest accrues on outstanding principal every month. The interest rate prior to consolidation is linked to the yield on the 3 month Treasury bill on May 30 each year, and is fixed for a year. Thus the reference rate, $\tilde{R}_{12k+i-1}$, in month $i$ of year $k$ is:

$$\tilde{R}_{12k+i-1} = \exp\left[4y^i (12k, 12k + 3/12)\right] - 1, \forall i = 0, ..., 11, k = 0, 1, 2, ...$$

The borrower pays an interest rate that depends on whether they are in repayment and whether they have consolidated. Extension of loan term without consolidation does not affect the interest rate. Before consolidation and repayment begins the student rate, $r_s$, is the lower of the reference rate plus 1.7 percent and the interest rate cap of 8.25 percent,

$$R_{S,t}^j = \min\left[\tilde{R}_t^j + 1.7\%, 8.25\%\right], \forall t < T^R \text{ and } t \leq t^c$$

After entering repayment but prior to consolidation, the student rate is the lower of the reference rate plus 2.3 percent and the interest rate cap:

$$R_{S,t}^j = \min\left[\tilde{R}_t^j + 2.3\%, 8.25\%\right], \forall t \geq T^R \text{ and } t \leq t^c$$

After consolidation, the student rate is fixed at the rate prevailing at the time of consolidation, $t^c$:

$$R_{S,t}^j = R_{S,t}^{j+k} \forall k > 0$$

The variable $P_t^j$ denotes the evolution of principal (prior to default) over time in each simulation $j$. Given an initial principal of $P_0^j = P_0$, principal evolves according to:

$$P_{i+1}^j = P_t^j \left[1 + r_{S,t}^j\right] - A_{t+1}^j$$

where

$$r_{S,t}^j = \left(1 + R_{S,t}^j\right)^{1/12} - 1$$

is the monthly compounding student rate. The prescribed monthly payment, $A_t^j$, depends on the loans status, and is based on amortizing the principal at the current interest rate over the remaining life of the loan:

$$A_{t+1}^j = \begin{cases} \frac{P_t^j r_{S,t}^j}{1 - \left(1 + r_{S,t}^j\right)^{-1}}, & t \geq T^R \\ 0, & t < T^R \end{cases}$$
In the direct program, the government’s cash flows on performing loans are the student loan payments less any administrative fees:

\[ A^j_t - fP^j_t \]

where \( f \) is the proportional administrative fee. The fee is 0.75 percent per annum in the benchmark calibration, reflecting typical servicing and other administrative costs. In default, the government recovers in proportion to the present value of remaining payments.

In the guaranteed lending program, the government cash flows are the quarterly payments to lenders – the SAP less any consolidation fee paid by lenders to the government – while the loan is in good standing, and the lump sum payment of outstanding principal and accrued interest in the event of default. We ignore administrative costs since they are largely borne by the guaranteed lender.

The quarterly SAP is the difference between the student rate and the 3-month commercial paper rate plus a spread, but has a floor of zero. We assume the annualized 3-month commercial paper rate, \( R_C \), tracks the t-bill rate with a 20 basis point spread:

\[ R^j_{C,t} = \exp \left[ 4y^j \left( 3k, 3k + 3/12 \right) \right] + .002 - 1, \forall t = 1, 2, ..., T \]

Absent default, the government cash flow in each month is the SAP less any consolidation fee paid from lenders to the government, or 1.05% of principal. We denote the net guarantee payment from the government while the loan is in good standing by \( G_j^t \):

\[
G_j^t = \begin{cases} 
- P^j_{3k} \max \left[ R^j_{C,3k} + 1.74\% - R^j_{5,3k}, 0 \right] / 4, & 3k < T^R \text{ and } 3k < T^C \quad \forall k = 0,1,2,... \\
- P^j_{3k} \max \left[ R^j_{C,3k} + 2.34\% - R^j_{5,3k}, 0 \right] / 4, & 3k \geq T^R \text{ and } 3k < t^C \quad \forall k = 0,1,2,... \\
- P^j_{3k} \left( \max \left[ R^j_{C,3k} + 2.64\% - R^j_{5,3k}, 0 \right] - 1.05\% \right) / 4, & 3k \geq t^C \quad \forall k = 0,1,2,... \\
0, & \text{otherwise}
\end{cases}
\]

In default, the government pays the outstanding principal, \( P^j_t \), to the lender, assumes the loan and recovers in proportion to the present value of the remaining outstanding payments. The default and recovery rates used in the calibration are described in section B.5.

Several additional factors affect the timing and magnitude of student loan cash flows, but we omit them from the analysis because we expect their impact on consolidation cost to be small. Voluntary prepayment has the effect of reducing the cost to the government by shortening effective loan maturity, while deferment and forbearance, by delaying payment, increase it. Subsidized Stafford loans offer a higher subsidy than so-called unsubsidized Stafford loans, because interest does not accrue while the student is in school. In the prospective cost estimates, this effects the principal balance at the time repayment begins, making it smaller than it otherwise would be. We assume all loans are unsubsidized loans in the analysis.

**Stochastic Rules Governing Borrower Behavior**
TBA – Describe rules for borrower consolidation, prepayment, defferment, grace and
default.

**Adjusting Discount Rates for Default Risk**

Under the CIR model, the risk neutral monthly compounded discount rate, $d_t$, for
default free but possibly interest rate contingent monthly cash flows is

$$d_t = \frac{1}{p(t, t + 1/12)} - 1 \forall t = 0, 1, 2, \ldots$$

In both the direct and guaranteed lending programs, the underlying payments
between parties are contingent on default. We adopt a parsimonious approach to default
that treats guaranteed and direct lending cash flows consistently.

We assume that default occurs with equal probability in each month until the
borrower completely repays the loan. Implicitly, we assume that the priced component of
default risk is orthogonal to the priced component of interest rate risk. Suppose there is a
pair of simple one-period securities traded in every period. The first is risk free offering
a certain payoff of $1 in one period’s time. The second is a risky claim that pays $1 if the
borrower does not default and $\alpha$ if the borrower does default. The fair price of the
default free claim is:

$$\frac{1}{1 + d_t}$$

With a constant monthly risk premium of $\pi$ and a default probability of $q$, the fair
price of the risky claim, $\varphi$, is:

$$\frac{1 - q(1 - \alpha)}{(1 + d_t)(1 + \pi)}$$

More conveniently, we can define state price deflators to value cash flows in $t+1$
paid if the borrower defaults:

$$\frac{h}{1 + d_t}$$

and if the borrower does not default:

$$\frac{1 - h}{1 + d_t}$$

where $h$ is the risk neutral probability of default:
Calibrating State Prices for Default Contingent Prices

Data from the NDSL suggests a cumulative default rate over the life of a typical Stafford loan of 15 percent. Default rates vary over the life of a loan, with the rate decreasing as the loan ages. Abstracting from the time pattern, an annual default rate of 2 percent is consistent with this cumulative experience. Hence, the quarterly default rate, \( q \), is set to .25 percent. Estimates from the NSLDS suggest a recovery rate on defaulted loans in the range 40 to 60 percent. We assume the mid-point of 50 percent in the computation of subsidy cost for the two programs.

Present Value of Program Cash Flows

To properly incorporate the state prices for default risk, it is convenient to consider a binomial representation of the components of cash flows. Figure A1 is a binomial tree representation of the cash flows for a two period student loan (ignoring administrative costs). Each up move in the tree indicates the borrower does not default, and each downward move corresponds to a borrower default. Rather than terminate the loan after default, to allow a stationary representation we assume that the borrower and lender agree to a new loan with payments reduced to fraction \( \alpha \) of the originally prescribed payments (reflecting failed collections and collection costs). That is, the lender recovers a lump sum proportional to the present value of remaining payments. The present value of loan cash flows using these assumptions is:

\[
\sum_{i=0}^{\infty} \frac{A_i}{\prod_{k=0}^{i-1} (1+d_k)(1+s)^i} = \frac{\pi + q(1-\alpha)}{(1+\pi)(1-\alpha)}
\]

where

\[
s = \frac{1+\pi}{1-q(1-\alpha)} - 1
\]

The variable \( s \) has an interpretation as the monthly credit spread, which depends on the rate of default (\( \pi \)), the rate of recovery (\( \alpha \)) and the risk premium for credit risk (\( q \)).
Assuming the administrative fee is paid only while the loan is not in default (only along the uppermost branches of the binomial tree) the present value of administrative fees is:

\[
\sum_{i=0}^{\infty} \frac{(1-h)^i fP_i^l}{\prod_{k=0}^{T-1} (1 + d_k)}
\]

Thus the present value of a direct loan is the difference between \( A \) and \( P \). The cash flows for the guarantee also have a binomial tree representation, as shown in Figure B2 for the two-period case.

These cash flows can be decomposed into two simpler binomial trees as displayed in Figure B3. The first is just the binomial tree for the student loan and the second is a residual that captures the net payments to lenders as described in section 5.3. Those lender payments have only two non-zero branches in each period because the loan is assumed to become federal property following default.
Valuing the two components of the using the risk neutral discount rates and probabilities of default gives the following present value of cash flows:

\[
\sum_{t=0}^{\infty} \frac{A_t}{\left(\prod_{k=0}^{t-1} (1+d_k)\right)(1+s)^t} - \sum_{t=0}^{\infty} \frac{\left(1-h\right)^t \left(G_t + A_t\right)}{\left(\prod_{k=0}^{t-1} (1+d_k)\right)} - \sum_{t=0}^{\infty} \frac{\left(1-h\right)^{t-1} hP_t}{\left(\prod_{k=0}^{t-1} (1+d_k)\right)}
\]